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COUPLED SIMULTANEOUS HEAT AND MASS TRANSFER IN MULTICOMPONENT TWO-PHASE MIXTURES

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A method is proposed for calculating the parameters of simultaneous heat and mass transfer in a multicomponent two-phase gas–liquid system, this method being based on solving the system of differential equations of convective heat transfer and convective diffusion.

An important item in research concerning heat- and mass-transfer processes is development of a theory for simultaneous heat and mass transfer in multicomponent two-phase mixtures. Particular attention is paid to solution of this problem as a coupled one.

A method of solving such problems will be outlined here on the example of heat and mass transfer in a multicomponent two-phase gas–liquid system which flows through a vertical channel in the descending parallel-flow mode.

Let the x axis run along a channel wall and the y axis run perpendiclar to it. The thermal diffusivity of each component and the coefficients of multicomponent diffusion are

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assumed to be independent of the temperature and the concentration at any time [1-3]. We will consider the case of steady-state heat and mass transfer with negligible resultant diffusional transfer of enthalpy within the volume of phases, with both liquid and gas flowing at mean rates, and with the parameters of both phases changing much slower in the longitudinal direction than in the transverse one. We will also assume that the dissipative term in the equation of convective heat transfer and thermodiffusional mass transfer can be disregarded [4], and that the film thickness changes neither in the longitudinal direction nor in the transverse one during the heat and mass transfer process. On this premise, then, the system of equations of convective mass transfer and heat transfer is

$$u_L \frac{\partial T_L}{\partial x} = \varkappa_L \frac{\partial^2 T_L}{\partial y^2}, \quad u_G \frac{\partial T_G}{\partial x} = \varkappa_G \frac{\partial^2 T_G}{\partial y^2},$$
$$u_L \frac{\partial \mathbf{C}_L}{\partial x} = [D_L] \frac{\partial^2 \mathbf{C}_L}{\partial y^2}, \quad u_G \frac{\partial \mathbf{C}_G}{\partial x} = [D_G] \frac{\partial^2 \mathbf{C}_G}{\partial y^2}. \tag{1}$$

The boundary conditions are stipulated in the form of the following relations at the channel entrance x = 0

$$T_L = T_{0_L}, \quad T_G = T_{0_G}, \quad \mathbf{C}_L = \mathbf{C}_{0L}, \quad \mathbf{C}_G = \mathbf{C}_{0G}, \tag{2}$$

at the pipe wall y = 0

$$T_L = T_{0_L}, \quad \mathbf{C}_L = \mathbf{C}_{0_L}, \tag{3}$$

and at the pipe axis y = R

$$T_G = T_{c_G}, \quad \mathbf{C}_G = \mathbf{C}_{0G}. \tag{4}$$

At the interphase boundary y = h_0 are satisfied the conditions of phase equilibrium, heat balance, and mass balance

$$T = T_L = T_G$$
$$\mathbf{C}_G = \ulcorner m \lrcorner \mathbf{C}_L + \mathbf{p}_1 T + \mathbf{p}_2, \tag{5}$$

$$q_L = q_G + \sum_{i=1}^{n} J_i \Delta \bar{H}_i, \tag{6}$$

where

$$J_i = J_{L\,i} = J_{Gi}, \quad i = 1, 2, \ldots, n. \tag{7}$$

With the aid of the identity $\sum_{i=1}^{n} J_i = 0$, one can transform condition (6) into

$$q_L = q_G + \sum_{i=1}^{n-1} J_i \Delta H_i, \text{ where } \Delta H_i = \Delta \bar{H}_i - \Delta \bar{H}_n. \tag{8}$$

Using Fourier's law and the generalized Fick's law, we obtain from expressions (7) and (8) the balance relations at the interphase boundary in the form

$$\lambda_L \frac{\partial T_L}{\partial y} = \lambda_G \frac{\partial T_G}{\partial y} + (\Delta \mathbf{H})^T [D_L] \frac{\partial \mathbf{C}_L}{\partial y}, \tag{9}$$

$$[D_L] \frac{\partial \mathbf{C}_L}{\partial y} = [D_G] \frac{\partial \mathbf{C}_G}{\partial y}. \tag{10}$$

The matrices $[D_G]$ and $[D_L]$ in the system of equations (1) contain coefficients of molecular diffusion for the vapors of mixture components. Expressions for the elements of both matrices can be derived from the molecular theory of gases [1-5]. An important property of these matrices is their reducibility to diagonal form [1]

$$[G]^{-1} [D_G] [G] = \ulcorner D'_G \lrcorner; \quad [L]^{-1} [D_L] [L] = \ulcorner D'_L \lrcorner. \tag{11}$$

The system of equations (1) with boundary conditions (2)-(5) and (9)-(10) is a coupled system of parabolic equations with boundary conditions of the fourth kind, thus a rather difficult one to solve. We therefore utilize properties (11) of the matrices $[D_G]$ and $[D_L]$ and transform the original system of equations into

$$u_L \frac{\partial T_L}{\partial x} = \varkappa_L \frac{\partial^2 T_L}{\partial y^2}, \quad u_G \frac{\partial T_G}{\partial x} = \varkappa_G \frac{\partial^2 T_G}{\partial y^2},$$
$$u_L \frac{\partial \mathbf{C}'_L}{\partial x} = \ulcorner D'_L \lrcorner \frac{\partial^2 \mathbf{C}'_L}{\partial y^2}, \quad u_G \frac{\partial \mathbf{C}'_G}{\partial x} = \ulcorner D'_G \lrcorner \frac{\partial^2 \mathbf{C}'_G}{\partial y^2}, \tag{12}$$

with

$$\mathbf{C}'_L = [L]^{-1}\mathbf{C}_L; \quad \mathbf{C}'_G = [G]^{-1}\mathbf{C}_G. \tag{13}$$

At x = 0 we have

$$T_L = T_{0L}, \quad T_G = T_{0G}, \quad \mathbf{C}'_L = \mathbf{C}'_{0L}, \quad \mathbf{C}'_G = \mathbf{C}'_{0G}, \tag{14}$$

where $C'_{0L} = [L]^{-1}C_{0L}$, $C'_{0G} = [G]^{-1}C_{0G}$.

At y = 0

$$T_L = T_{0L}, \quad \mathbf{C}'_L = \mathbf{C}'_{0L}. \tag{15}$$

At y = R

$$T_G = T_{0G}, \quad \mathbf{C}'_G = \mathbf{C}'_{0G}. \tag{16}$$

At $y = h_0$

$$T_L = T_G = T, \quad \mathbf{C}'_G = [R_1]\mathbf{C}'_L + \mathbf{p}'_1 T + \mathbf{p}'_2,$$
$$[R_2]\ulcorner D'_L \lrcorner \frac{\partial \mathbf{C}'_L}{\partial y} = \ulcorner D'_G \lrcorner \frac{\partial \mathbf{C}'_G}{\partial y}, \quad \lambda_L \frac{\partial T_L}{\partial y} = \lambda_G \frac{\partial T_G}{\partial y} + \mathbf{r}_1^T \frac{\partial \mathbf{C}'_L}{\partial y}, \tag{17}$$

where

$$\mathbf{p}'_1 = [G]^{-1}\mathbf{p}_1; \quad \mathbf{p}'_2 = [G]^{-1}\mathbf{p}_2; \quad \mathbf{r}_1^T = (\Delta\mathbf{H})^T[D_L][L];$$
$$[R_1] = [G]^{-1}\ulcorner m \lrcorner [L]; \quad [R_2] = [G]^{-1}[L]. \tag{18}$$

Let $y_1 \equiv y$ in the $0 \le y \le h_0$ range and $y_2 \equiv y$ in the $h_0 \le y \le R$ range, whereupon we introduce dimensionless coordinates η_1 and η_2 according to the relations

$$\eta_1 = 1 - \frac{y_1}{h_0}, \quad \eta_2 = 1 - \frac{R - y_2}{R - h_0}.$$

Then the system of relations (12)-(17) finally becomes

$$\frac{\partial T_L}{\partial x} = \gamma_{TL} \frac{\partial^2 T_L}{\partial \eta_1^2}, \quad \frac{\partial T_G}{\partial x} = \gamma_{TG} \frac{\partial^2 T_G}{\partial \eta_2^2},$$
$$\frac{\partial \mathbf{C}'_L}{\partial x} = \ulcorner \gamma_L \lrcorner \frac{\partial^2 \mathbf{C}'_L}{\partial \eta_1^2}, \quad \frac{\partial \mathbf{C}'_G}{\partial x} = \ulcorner \gamma_G \lrcorner \frac{\partial^2 \mathbf{C}'_G}{\partial \eta_2^2}, \tag{19}$$

where

$$\gamma_{TL} = \frac{\varkappa_L}{u_L h_0^2}; \quad \gamma_{TG} = \frac{\varkappa_G}{u_G (R - h_0)^2}; \quad \gamma_{Li} = \frac{D_{Li}}{u_L h_0^2}; \quad \gamma_{Gi} = \frac{D_{Gi}}{u_G (R - h_0)^2}.$$

At x = 0 we have

$$T_L = T_{0L}, \quad T_G = T_{0G}, \quad \mathbf{C}'_L = \mathbf{C}'_{0L}, \quad \mathbf{C}'_G = \mathbf{C}'_{0G}, \tag{20}$$

at $\eta_1 = 1$

$$T_L = T_{0L}, \quad \mathbf{C}'_L = \mathbf{C}'_{0L}, \tag{21}$$

at $\eta_2 = 1$

$$T_G = T_{0G}, \quad \mathbf{C}'_G = \mathbf{C}'_{0G},$$

and at $\eta_1 = \eta_2 = 0$

$$T_L = T_G = T, \quad \mathbf{C}'_G = [R_1]\,\mathbf{C}'_L + \mathbf{p}'_1 T + \mathbf{p}'_2, \tag{22}$$

$$\lambda_G \frac{\partial T_G}{\partial \eta_2} = -\frac{R-h_0}{h_0}\lambda_L \frac{\partial T_L}{\partial \eta_1} + \frac{R-h_0}{h_0}\mathbf{r}_1^T \frac{\partial \mathbf{C}'_L}{\partial \eta_1}, \tag{23}$$

$$\ulcorner D'_G \lrcorner \frac{\partial \mathbf{C}'_G}{\partial \eta_2} = -\frac{R-h_0}{h_0}[R_2]\ulcorner D'_L \lrcorner \frac{\partial \mathbf{C}'_L}{\partial \eta_1}. \tag{24}$$

We will solve Eqs. (19) in the boundary-layer approximation, assuming that the main changes in concentrations C'_{Li}, C'_{Gi} and temperatures T_L, T_G of components occur within layers adjacent to the interphase boundary, considering also that those functions are bounded

$$T_L = a_1 + b_1 \int_0^{w_1} \exp(-\Psi^2)\,d\Psi, \quad T_G = a_2 + b_2 \int_0^{w_2} \exp(-\Psi^2)\,d\Psi,$$

$$C'_{Li} = A_{1i} + B_{1i} \int_0^{z_{1i}} \exp(-\Psi^2)\,d\Psi, \quad C'_{Gi} = A_{2i} + B_{2i} \int_0^{z_{2i}} \exp(-\Psi^2)\,d\Psi, \tag{25}$$

where

$$w_1 = \frac{\eta_1}{2\sqrt{\gamma_{TL} x}}; \quad w_2 = \frac{\eta_2}{2\sqrt{\gamma_{TG} x}}; \quad z_{1i} = \frac{\eta_1}{2\sqrt{\gamma_{Li} x}};$$

$$z_2 = \frac{\eta_2}{2\sqrt{\gamma_{Gi} x}}; \quad i = 1, 2, \ldots, n-1.$$

For determining the integration constants a_1, a_2, b_1, b_2, A_{1i}, A_{2i}, B_{1i}, B_{2i}, we will use initial and boundary conditions (20), (22)-(24), considering that

$$T_{0L} = a_1 + b_1 \frac{\sqrt{\pi}}{2}, \quad T_{0G} = a_2 + b_2 \frac{\sqrt{\pi}}{2},$$

$$\mathbf{C}'_{0L} = \mathbf{A}_1 + \mathbf{B}_1 \frac{\sqrt{\pi}}{2}, \quad \mathbf{C}'_{0G} = \mathbf{A}_2 + \mathbf{B}_2 \frac{\sqrt{\pi}}{2}, \tag{26}$$

$$a_1 = a_2 = a, \quad \mathbf{A}_2 = [R_1]\,\mathbf{A}_1 + \mathbf{p}'_1 a + \mathbf{p}'_2,$$

$$b_2 = -\sqrt{\frac{u_L}{u_G}}\,[r_3 b_1 - r_4 \mathbf{r}_1^T (\ulcorner D'_L \lrcorner)^{-0.5}\mathbf{B}_1], \quad \mathbf{B}_2 = -\sqrt{\frac{u_L}{u_G}}\,[R_3]\,\mathbf{B}_1, \tag{27}$$

where

$$r_3 = \frac{\sqrt{\varkappa_G}\,\lambda_L}{\sqrt{\varkappa_L}\,\lambda_G}; \quad r_4 = \frac{\sqrt{\varkappa_G}}{\lambda_G}; \quad [R_3] = (\ulcorner D'_G \lrcorner)^{-0.5}\,[R_2]\,(\ulcorner D'_L \lrcorner)^{0.5} \tag{28}$$

by virtue of the assumptions regarding the functions C'_{Li}, C'_{Gi}, T_L, T_G. The solution to the system of algebraic scalar-vector equations (26)-(27) will be obtained in the form

$$b_1 = \frac{2}{\sqrt{\pi}}\varphi \frac{\sqrt{\frac{u_L}{u_G}}\, r_4 \mathbf{r}_1^T (\ulcorner D'_L \lrcorner)^{-0.5}\,[\Omega]\,(\vec{\boldsymbol{\omega}} + \mathbf{p}'_1 T_{0L} + \mathbf{p}'_2) + \Delta T_0}{1 + \varphi \sqrt{\frac{u_L}{u_G}}\, r_4 \mathbf{r}_1^T (\ulcorner D'_L \lrcorner)^{-0.5}\,[\Omega]\,\mathbf{p}'_1}, \tag{29}$$

$$b_2 = b_1 - \frac{2}{\sqrt{\pi}}\Delta T_0, \quad a_1 = a_2 = T_{0L} - \frac{\sqrt{\pi}}{2} b_1,$$

$$\mathbf{B}_1 = [\Omega]\left\{\frac{2}{\sqrt{\pi}}(\vec{\boldsymbol{\omega}} + \mathbf{p}_1' T_{0\mathrm{L}} + \mathbf{p}_2') - \mathbf{p}_1 b_1\right\}, \quad \mathbf{B}_2 = -\sqrt{\frac{u_{\mathrm{L}}}{u_G}}\,[R_3]\mathbf{B}_1,$$

$$\mathbf{A}_1 = \mathbf{C}_{0\mathrm{L}}' - \frac{\sqrt{\pi}}{2}\mathbf{B}_1, \quad \mathbf{A}_2 = \mathbf{C}_{0G}' - \frac{\sqrt{\pi}}{2}\mathbf{B}_2, \tag{30}$$

where

$$\varphi = \left(\sqrt{\frac{u_{\mathrm{L}}}{u_G}}\, r_3 + 1\right)^{-1}; \quad \Delta T_0 = T_{0\mathrm{L}} - T_{0G};$$

$$[\Omega] = \left(\sqrt{\frac{u_{\mathrm{L}}}{u_G}}\,[R_3] + [R_1]\right)^{-1}; \quad \vec{\boldsymbol{\omega}} = [R_1]\,\mathbf{C}_{0\mathrm{L}}' - \mathbf{C}_{0G}'. \tag{31}$$

Expressions (29)-(31) together with expressions (25) make it possible to determine the temperatures T_{L}, T_G and to convert the concentrations C'_{L}, C'_G for any values of the coordinates.

It is now necessary to change back to real values of concentrations C_{L}, C_G. Such a transition can be made with the aid of relations (13).

We introduce the notation

$$S_{T\mathrm{L}}(x, \eta_1) = \int_0^{\omega_1} \exp(-\Psi^2)\,d\Psi, \quad S_{TG}(x, \eta_2) = \int_0^{\omega_2} \exp(-\Psi^2)\,d\Psi,$$

$$S_{\mathrm{L}i}'(x, \eta_1) = \int_0^{z_{1i}} \exp(-\Psi^2)\,d\Psi, \quad S_{Gi}'(x, \eta_2) = \int_0^{z_{2i}} \exp(-\Psi^2)\,d\Psi.$$

Then expressions (25) with the aid of notation (30) become

$$T_{\mathrm{L}}(x, \eta_1) = T_{0\mathrm{L}} + \left(S_{T\mathrm{L}}(x, \eta_1) - \frac{\sqrt{\pi}}{2}\right) b_1, \tag{32}$$

$$T_G(x, \eta_2) = T_{0G} + \left(S_{TG}(x, \eta_2) - \frac{\sqrt{\pi}}{2}\right) b_2, \tag{33}$$

$$\mathbf{C}_{\mathrm{L}}'(x, \eta_1) = \mathbf{C}_{0\mathrm{L}}' + \left(\ulcorner S_{\mathrm{L}}' \lrcorner - \frac{\sqrt{\pi}}{2} \ulcorner I \lrcorner\right) \mathbf{B}_1, \tag{34}$$

$$\mathbf{C}_G'(x, \eta_2) = \mathbf{C}_{0G}' + \left(\ulcorner S_G' \lrcorner - \frac{\sqrt{\pi}}{2} \ulcorner I \lrcorner\right) \mathbf{B}_2. \tag{35}$$

Now using relations (13) we obtain from expressions (34) and (35)

$$\mathbf{C}_{\mathrm{L}} = [L]\,\mathbf{C}_{\mathrm{L}}' = \mathbf{C}_{0\mathrm{L}} + \left([S_{\mathrm{L}}(x, \eta_1)] - \frac{\sqrt{\pi}}{2} \ulcorner I \lrcorner\right) [L]\,\mathbf{B}_1, \tag{36}$$

$$\mathbf{C}_G = [G]\,\mathbf{C}_G' = \mathbf{C}_{0G} + \left([S_G(x, \eta_2)] - \frac{\sqrt{\pi}}{2} \ulcorner I \lrcorner\right) [G]\,\mathbf{B}_2, \tag{37}$$

where

$$[S_{\mathrm{L}}(x, \eta_1)] = [L]\ulcorner S_{\mathrm{L}}' \lrcorner [L]^{-1}; \quad [S_G(x, \eta_2)] = [G]\ulcorner S_G' \lrcorner [G]^{-1}.$$

Upon performing the transformations

$$[L]\,\mathbf{B}_1 = [L]\,[\Omega]\,([G]^{-1}[G])\left\{\frac{2}{\sqrt{\pi}}(\vec{\boldsymbol{\omega}} + \mathbf{p}_1' T_{0\mathrm{L}} + \mathbf{p}_2') - \mathbf{p}_1' b_1\right\} = \frac{2}{\sqrt{\pi}}[\Phi]\,(\Delta\mathbf{C}_0)_T, \tag{38}$$

$$[G]\,\mathbf{B}_2 = -[G]\sqrt{\frac{u_L}{u_G}}\,[R_3]\,\mathbf{B}_1 = -\sqrt{\frac{u_L}{u_G}}\,[D_G]^{-0.5}\,[D_L]^{0.5}\,[L]\,\mathbf{B}_1 = \frac{2}{\sqrt{\pi}}\left(\ulcorner m \lrcorner\,[\Phi] - \ulcorner I \lrcorner\right)(\Delta\mathbf{C}_0)_T,$$

where

$$[\Phi] = \left(\sqrt{\frac{u_L}{u_G}}\,[D_G]^{-0.5}\,[D_L]^{0.5} + \ulcorner m \lrcorner\right)^{-1};$$

$$(\Delta\mathbf{C}_0)_T = \ulcorner m \lrcorner\,\mathbf{C}_{0L} - \mathbf{C}_{0G} + \mathbf{p}_1 T_{0L} + \mathbf{p}_2 - \frac{\sqrt{\pi}}{2}\,\mathbf{p}_1 b_1, \tag{39}$$

and inserting the result into expressions (36) and (37), we obtain

$$\mathbf{C}_L(x,\ \eta_1) = \mathbf{C}_{0L} + \left(\frac{2}{\sqrt{\pi}}\,[S_L(x,\ \eta_1)] - \ulcorner I \lrcorner\right)[\Phi]\,(\Delta\mathbf{C}_0)_T, \tag{40}$$

$$\mathbf{C}_G(x,\ \eta_2) = \mathbf{C}_{0G} + \left(\frac{2}{\sqrt{\pi}}\,[S_G(x,\ \eta_2)] - \ulcorner I \lrcorner\right)\left(\ulcorner m \lrcorner\,[\Phi] - \ulcorner I \lrcorner\right)(\Delta\mathbf{C}_0)_T. \tag{41}$$

An analogous procedure will reduce expression (29) to

$$b_1 = \frac{2}{\sqrt{\pi}}\,\varphi\,\frac{\sqrt{\frac{u_L}{u_G}}\,\frac{\sqrt{\varkappa_G}}{\lambda_G}\,(\Delta\mathbf{H})^T\,[D_L]^{0.5}\,[\Phi]\,(\ulcorner m \lrcorner\,\mathbf{C}_{0L} - \mathbf{C}_{0G} + \mathbf{p}_1 T_{0L} + \mathbf{p}_2) + \Delta T_0}{1 + \varphi\sqrt{\frac{u_L}{u_G}}\,\frac{\sqrt{\varkappa_G}}{\lambda_G}\,(\Delta\mathbf{H})\;[D_L]^{0.5}\,[\Phi]\,\mathbf{p}_1}, \tag{42}$$

and thus to one not containing intermediate quantities, more convenient for direct calculations.

The obtained expressions (32)-(33) and (40)-(41) make it possible, therefore, to determine the temperature fields and the concentration fields in both liquid and gaseous phases, namely the temperature and the concentration at any point $(x,\ \eta_1)$ or $(x,\ \eta_2)$ in the regions $x \geqslant 0$, $0 \leq \eta_1 \leq 1$, $0 \leq \eta_2 \leq 1$.

Knowing both temperature and concentration distributions in each phase, one can determine heat and mass fluxes of the mixture components at the interphase boundary. For this purpose one has to differentiate expressions (32) and (34) at the point $\eta_1 = 0$, which yields

$$\left.\frac{\partial T_L}{\partial \eta_1}\right|_{\eta_1 = 0} = \frac{b_1}{2\sqrt{\gamma_{TL} x}}, \quad \left.\frac{\partial C'_{Li}}{\partial \eta_1}\right|_{\eta_1 = 0} = \frac{B_{1i}}{2\sqrt{\gamma_{Li} x}}, \quad i = 1,\ 2,\ \ldots,\ n-1.$$

Then the heat flux is

$$q = \frac{\lambda_L}{2}\sqrt{\frac{u_L}{\varkappa_L x}}\;b_1,$$

and the mass flux is

$$\mathbf{J}' = \frac{1}{2}\sqrt{\frac{u_L}{x}}\;(\ulcorner D'_L \lrcorner)^{0.5}\mathbf{B}_1,$$

with B_1 and b_1 defined by expressions (30) and (42), respectively.

A change to real fluxes J, with the aid of the obtained expression (38) for $[L]B_1$ yields

$$\mathbf{J} = [L]\,\mathbf{J}' = \sqrt{\frac{u_L}{\pi x}}\;[D_L]^{0.5}\,[\Phi]\,(\Delta\mathbf{C}_0)_T,$$

with $[\Phi]$ and $(\Delta C_0)_T$ defined by expressions (39).

Equivalent expressions for the fluxes can be obtained also from Eqs. (33) and (40) for the gaseous phase.

NOTATION

T, temperature of the mixture; C, vector of molar concentrations of the mixture components; C', vector of concentrations converted according to relations (13); [D], matrix of multicomponent diffusion coefficients; $\lceil D' \rfloor$, a diagonal matrix of eigenvalues of matrix [D]; [G] and [L], fundamental matrices for the gaseous component and the liquid component, respectively; λ, thermal conductivity; κ, thermal diffusivity; q, heat flux; J, vector of diffusion fluxes of the mixture components; $\lceil m \rfloor$, p_1, p_2, parameters in the equilibrium relation (5); $\Delta \tilde{H}_i$, difference between molar enthalpies of the i-th component in the gaseous phase and in the liquid phase, respectively, carried by its mass flux across the interphase boundary; n, number of mixture components; h_0, thickness of the liquid film; R, pipe radius; u, velocity of the phases in directional motion; t, x, y, space coordinates; η_1, η_2, dimensionless coordinates; and $\lceil I \rfloor$, unit matrix. Subscripts i refers to the i-th component; L, liquid phase; G, gaseous phase; and 0, value of a quantity at the boundary.

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CRITERION FOR THE BREAKUP OF LIQUID DROPS AND JETS

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UDC 532.529

The conditions of the breakup of liquid drops and jets are determined using wave theory.

The breakup of liquids is accomplished pneumatically, in particular, for the intensification of physicochemical processes in engineering. The mechanism of this process has been studied in many investigations, much of which has been systematized in [1-3]. It has been established that irregularities of the wave type develop on the surface of a liquid with the motion of a gas stream relative to it. These travel and increase in size, separating from the liquid surface and being converted into drops of smaller size than the initial volume of liquid. Since no significant difference in the conditions of liquid breakup is noted with variation of the position of the gas–liquid interface in space, it can be assumed that the waves have a capillary nature, and the theory of the development of these waves at a gas–liquid interface [4] can be used.

Let us assume that capillary waves develop on the surface of a volume of liquid at its frontal point when a gas stream impinges on it. Their amplitudes grow with time and over the period τ_{gr} they become comparable with the wavelength $\alpha \approx \lambda$, and according to [4] this leads to separation of the wave from the surface of the liquid, i.e., to the breakup of its original volume. Since the waves move over the surface of the volume of liquid, it is obvious that such breakup becomes possible if the growth time of at least one wave is less than the time τ_{mo} of its motion over the surface of the volume. On the other hand, it is necessary that the length of at least one wave be less than the characteristic size ℓ of the volume of liquid being broken up.

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